Nonlinearity Index And Structural Analysis–based System Identification Using Modal Assurance Criterion (MAC)

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Abstract-It is well known that all the dynamic system has some level of nonlinearity. This nonlinearity is caused from numerous sources that can be classified differently (e.g. material, geometry and contact). In this work a relative nonlinearity index is defined through vector correlation by expanding the existing methods and through the introduction of finite element analysis (FEA). Steels beams are investigated to determine its non-linear behaviour while considering the % variation in young's modulus 'E' in one case and variations in E along with the presence of a crack in another case. Severity of material assurance criteria (MAC) is also tried to find out by establishing a linear reference set point for both cases. This research acknowledge on the basis of the results of damage indices that the non linearity is maximum deviated towards the crack presence with respect to change in material properties (E).

Keywords-Non-linearity, MAC, Damage index

I. INTRODUCTION

In mechanical engineering and structure beams made of Aluminium and steel play an important role. For home construction the type of timber beam are used for the said purpose. In case of transverse loading, loads are applied perpendicular to the beam axis which causes bending and shear in the structure. When the axial forces are produced in the beam it means that the load is not applied perpendicular to the beam axis. Beam structures fails due to the damage present in the structure. For the detection of damage various techniques has been used to avoid the failure of the structure. In structure analysis procedure, change in natural frequencies is one of the common damage identification methods. When the damage present in the structure we normally observed that the stiffness along with the natural frequencies of the structure are reduced. By using this technique the best advantage is that the frequency of the system can quickly and easily be measured. Moreover for the purpose of resonant frequencies experimental techniques are used which are the basically vibration dimension techniques. Doebling et al. [I], in the first article proposed that with the help of vibration measurement technique, damage can be detected which was written by Lifshitz and Roterm [ii]. For the identification of damage in the elastomers they used the change in the natural frequencies along with the change in the modulus of elasticity. Hearn and Testa [ii] also elaborated that the ratio of the change in frequencies between two modes i and j is independent on the MAC severity but depends only on the crack position. This result shows the detection of damage position in the structure. Hasan [iii] elaborated this phenomenon for the cracked beam in elastic region. Many researchers have adopted the procedure to identify the damage in the structure with the help of changes in natural frequencies. Salawu collects massive literature on damage detection through varying frequencies [iv]. Some researchers [v-xv] showed the comparison between the natural frequencies of the damped and undamped structures. Nikolakopoulos and Papadopoulos [xvi] gave the idea that the first two Eigen frequencies depend on the crack depth and location which are to be represented in contour graph. Yang et al. [xvii] used the technique to detect the damage in simply supported beam with the help of change in natural frequencies method and the intersection of the plot lines which are obtained from different modes of vibration. In simply supported beam structure, it is observed that the point which are the intersection point of the three plot lines are accordingly to the estimated frequency progress and or accordingly factors due to the absence of damage which indicates not only one crack depth but also indicates two similar crack location because the structure are symmetric in simply supported beam structure [xiii-xiv]. Dong et al.[xviii] gave an idea of the development of mode shape to check the behaviour of the damaged structures because when the damage is present in the structure then its behaviour is non-uniqueness.Swamidas et al.[xix] gave a new idea about the mass attached to the structure which are to be examined and the mass should be unsymmetrical or off centre to minimize the symmetrical solution of the examined structure. Sinou [xx] explained by performing two tests on the structure by adding mass at different location for the identification of crack depth and the crack location. Kyriazoglou et al. [xxi] found new techniques for detecting the damage in the composite material with the help of measuring the specific damping capacity. Panteliou et al. [xxii] observed that with the increments of damping factor, crack depth is also increased. They also observed that the identification of the cracks with the help of change in damping factors is not sensitive to the boundary conditions and that's why it has the advantage in comparison to the change in natural frequencies technique. Gladwell and Morassi [xxiii] did research for the vibrating thin rod due to the damage effect on the nodes. They elaborated that the nodes of each mode shape moves along the direction of the damage. The node which located to left of the damage moves towards the right and the nodes which located to the right of the damage which is on the undamaged structure, moves towards the left [xxv, xxvi].Damage detection basically depends upon the study of the modal characteristics (i.e natural frequencies changes and mode shapes) which observed by Adams et al. [xxiv]. In this analysis they observed that only superior modes are used for the identification of damage. Natke and Cempel [xxvii] detected the damage in the cable stayed steel bridge by using the changes in mode shapes and Eigen frequencies. Law and Zhu [ii,viii] observed that when the deflection in the mode shapes increases then the damage in the structure also increases so the deflection in the mode shape can be a very important indicator for the identification of the damage. Doebling et al.[ii,iii] and West [v, vi] briefly discussed about the information of mode shape for detecting the damage in the structure without any use of finite element model. For the correlation of mode shape, a modal assurance criterion (MAC) is used for this purpose. For the comparison of i and j mode shapes MAC is used. The value of MAC is in between 0 and 1 [vii]. For the comparison of mode shapes, MAC basically uses the orthographic properties of these mode shapes. If the value of MAC is one then it indicates that two structures have the same mode shapes. If the value of MAC is zero then it indicates that the mode shapes are dissimilar. So the low MAC value can be used as the damage indication in the specimen. Srinivasan and Kot [v, viii] observed that the variation in MAC value are more responsive as compared to the change in natural frequencies of the given structure. Although the MAC show the visible difference between two data sets and also used for the detection of damage present in the structure but the main disadvantage is that it does not give information about the location of the damage where damage present in the structure. MAC is basically used for detecting the small crack present in the structure [ix, x]. Other technique such as frequency response is not used for the small cracks because it does not indicate the small cracks.

II. Modal assurance Criterion (MAC)

Modal assurance criterion (MAC) is a scalar quantity which indicates degree of consistency between any two sets of mode shapes [iv]. Its governing equation can be written as:

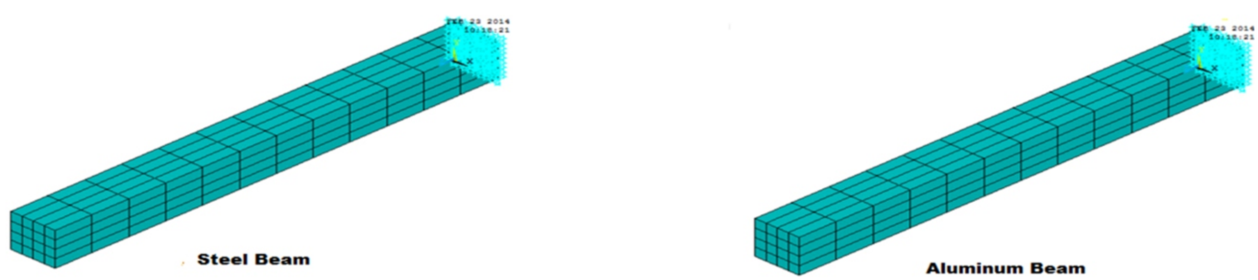
$$MAC = \frac{|(\Phi_1)^{\mathrm{T}}(\Phi_2)|^2}{((\Phi_2)^{\mathrm{T}}(\Phi_2))((\Phi_1)^{\mathrm{T}}(\Phi_1))}$$

where:

Φ_1= First Modal Vector,
Φ_2= Second Modal Vector

III. Finite Element Modelling of Beam

In order to carry out this study a 3-dimensional beam has been modelled in commercially available software ANSYS. Two kind of materials, steel and Aluminium, have been utilized for comparison purposes in which the Aluminium beam is consider to be a reference beam for analysis regarding variations in modulus of elasticity values in indexing study. The Fig.1 (a&b) shows the FEA model of two beams which we considered in this research.



(a) (b)

Fig. 1. Finite element model of steel and aluminium beams

In this research we will study the plane beam with a four-element finite element method. The elements features are shown in Table I. Then the results which are obtained from the FEM model are compared with the exact solution that will be obtained from the discontinuity functions. As in the FEM technique, when the number of elements increased then the solution obtained from the FEM model will be nearly closed to the solution which are obtained from the exact model. So increases the number of elements in FEM model the results will be more accurate and near to the exact model. Two beam structure made of aluminium and steel is considered in this research which are constrained at one end and other ends are free like cantilever beam structure. The geometric and material properties of both beams re represented in Table II and Table III respectively.

TABLE I
FEA BASED ELEMENTS FEATURES

Element Type	Solid 186
Element Shape	Quadrilateral
Element Behaviour	Plane Stress
Constrained Locations	x-axis

TABLE II
GEOMETRIC PROPERTIES OF THE BOTH BEAMS

Height of Beam	20mm
Width of Beam	20 mm
Length of Beam	200 *mm*
Number of Elements	4

TABLE III
MATERIAL PROPERTIES OF THE BEAMS

Material Properties	Aluminium	Steel
Density (kg/mm^3)	2800E-9	8000 E-9
Poisson's Ratio	0.28	0.3
Elasticity (N/mm^2)	71.7E3	200E3

IV. CRACK MODELLING

Crack starting and propagation in the beam structure basically change the overall response of the beam structure because when the crack present in the beam structure basically it reduces the stiffness of that structure so when the stiffness decreases then the system goes to the failure side. Following techniques are used in ANSYS to initiate the crack in the structure.

(a) Material removal technique at the crack location
(b) By attaching the lumped mass at crack location
(c) Periodic reduction in Young's Modulus 'E
(d) Fatigue or Hairline crack

In this research hairline crack or fatigue crack technique has been utilized for generation of crack. Crack depth is basically a ratio between the crack length (a) and the thickness (d) and the crack position is defined as the ratio between the crack distances from the base (b) of the structure to the length of that structure (I) as shown in Fig. 02.

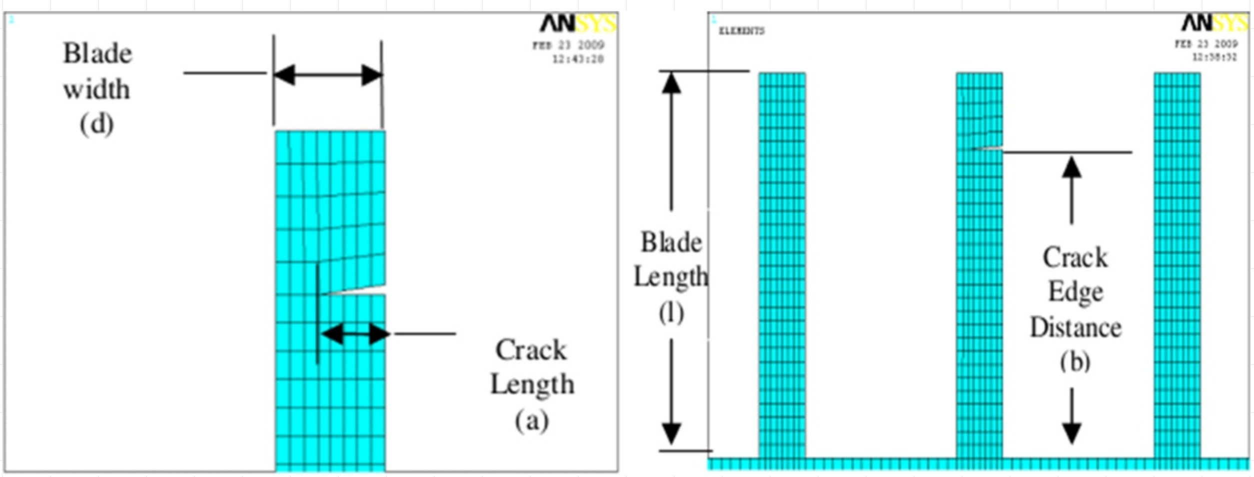


Fig. 2. FEA based crack features with respect to structure dimensions

In this research work,aluminium beam is taken as a reference beam and steel structure parameters are changed in term of modulus of elasticity and then model analysis performed on each structure to compare the result of these structures. Then compare the mode shapes and natural frequencies of these structures to examine the behaviour of these structures since geometry of these two beams are the same so only the modulus of elasticity is changed for the steel beam and examined the model characteristic of each beam. So after examining the results according to these changes, crack has been introduced in the steel beam but aluminium beam is still taken as the reference beam. So the total length of the steel beam is 200mm and width is 20mm so the crack has been introduced in the beam at the location of 100mm and the depth about 10mm so the crack depth is found 0.5.

V. Research Methodology

Aluminium beam is taking as the reference beam and steel beam changes with respect to its modulus of elasticity.± 1% increment, .± 5% increment,.± 10% increment,.± 25% increment,.+ 100% increment,.+ 200% increment, .+ 300% increment,.+ 400% increment, and + 500% increment. Then meshing will be done on these models. Then hairline or fatigue crack has been introduced in the steel beam along with the same increment in the modulus of elasticity in the steel beam. After this, model analysis performed on these models then from this analysis we extract the twenty mode shape from each model then against each mode shape we calculate the frequency data and vector data then these vector data solve through the Mat lab. The flow chart which is used in this research is shown in figure 03. After this MAC plot is obtained with the help of MATLAB which show the correlation between the mode shapes of different models. The damage index is calculated by comparing the MAC plot and auto MAC in each case.

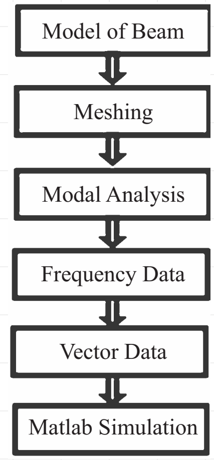


Fig. 3. Research sequence flow chart

VI. Modal Analysis

Using the model analysis we determine the model parameters i.e. mode shapes, natural frequencies of any system. So model analysis is performed for the first twenty mode shapes on ANSYS to observe the structure vibration and to make the comparison of natural frequencies between reference model (Fig. 04) and the steel model having crack(Fig. 06) and without crack (Fig. 05) along with the changes in modulus of elasticity. We extract the twenty mode shapes of each model and observe the behaviour of the beam structure. These are the sample of mode shapes.

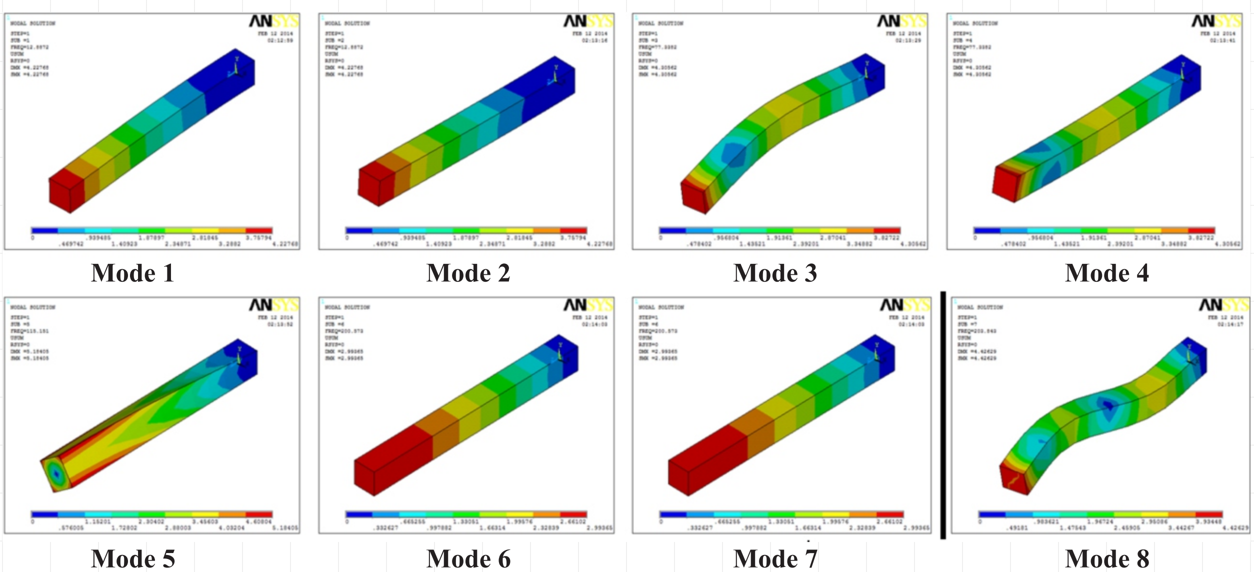


Fig. 4. Mode shapes (Reference model) of Aluminium Beam

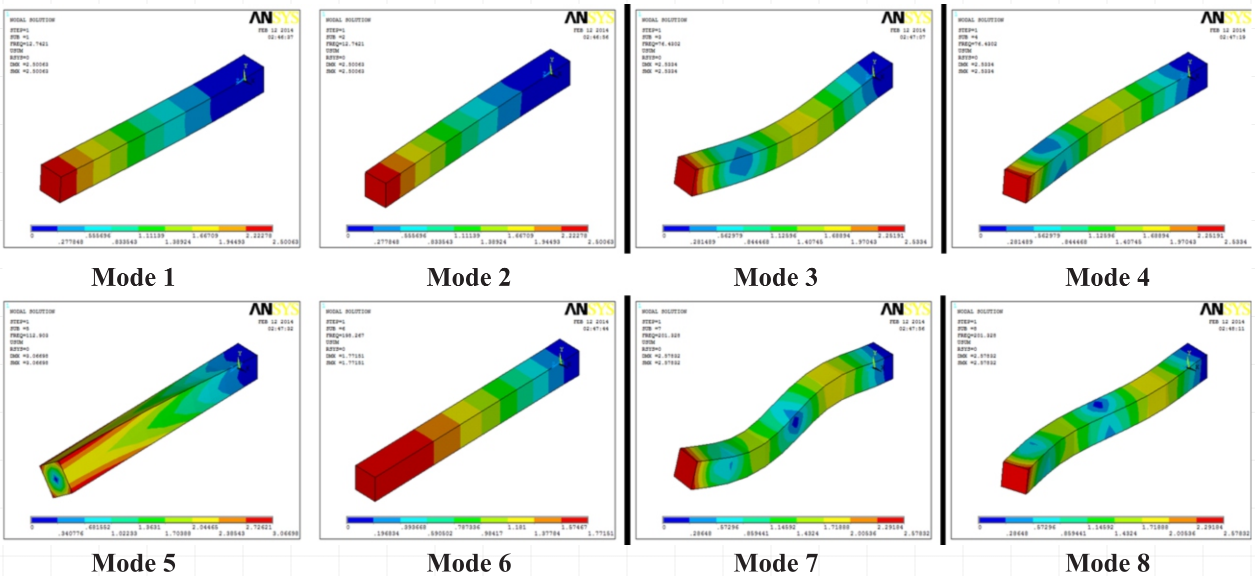


Fig. 5. Mode shapes of Steel Beam without crack

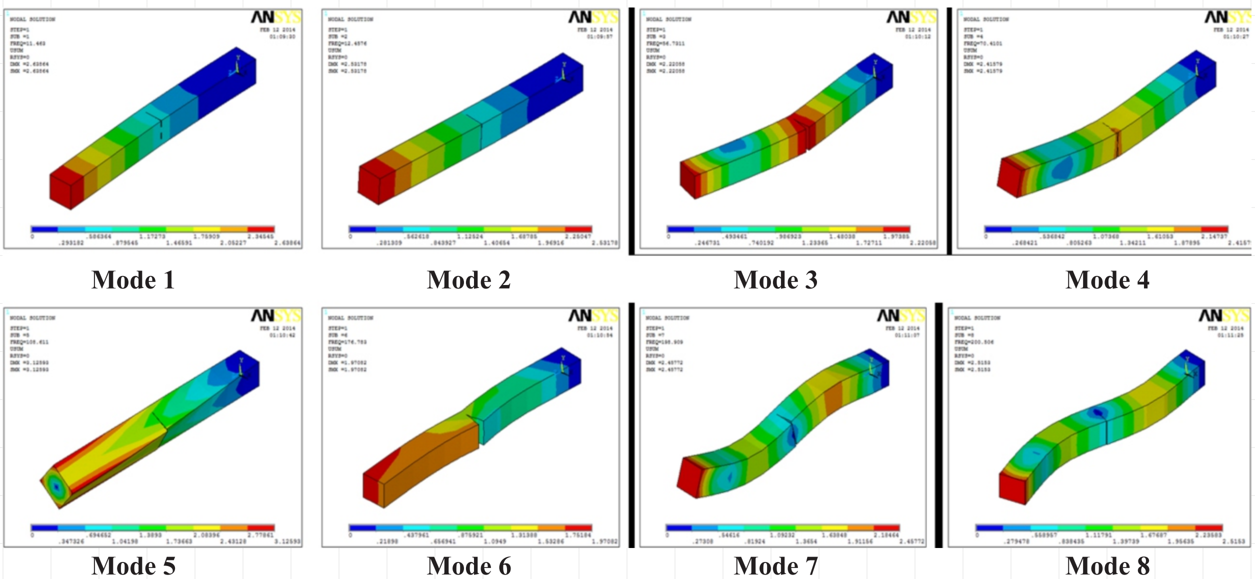


Fig. 6. Mode shapes of steel beam with crack

VII. Damage Index

A well-established, frequently used technique in fracture mechanics field named MAC, Model Assurance Criteria, is tried to use in this research work to correlate the shifting of vector data extracted from resultants of Cartesian geometries of modes shapes from material properties to varying mechanics of subjected material.

A successful design hypothesis is concluded by using the elaborated concept of DI (Damage Indices), a comparative examination of frobenius norm (solitary point of extensive data sheet of MACs), is efficiently applied to detect the dependence of material behavior against its inherent properties with and without presence of cracks.

For the description of modal assurance criteria and Auto Mac plot, two different cases are considered for the beam structure in this research work.Case-1, without crack model of Aluminium and steel but modulus of elasticity changes in steel model and in case-2 the crack has been introduced in the steel model up to 0.5 crack depth and crack location is also 0.5 along with the percentage variation in the modulus of elasticity of the steel structure but in both cases aluminium model is taken as the reference model. First twenty mode shapes are considered in both cases and then MAC and Auto Mac plots for correlating the mode shapes in each case. Auto Mac plot (Fig. 7) is taken in between the aluminium and steel models which is taken as the reference model and MAC plot (Fig. 8) is in between the aluminium and steel models having percentage variation in the modulus of elasticity.

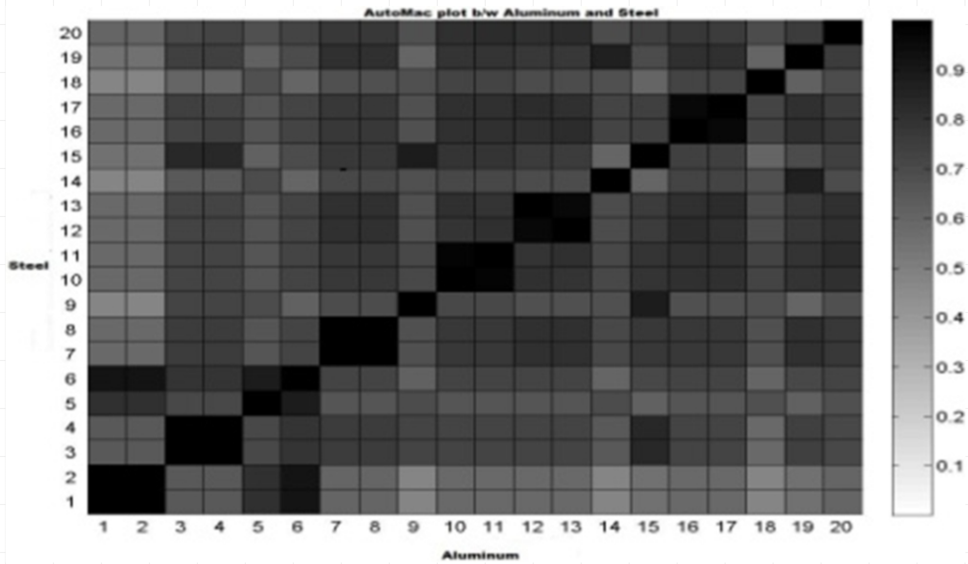


Fig. 7. Reference or AutoMac plot

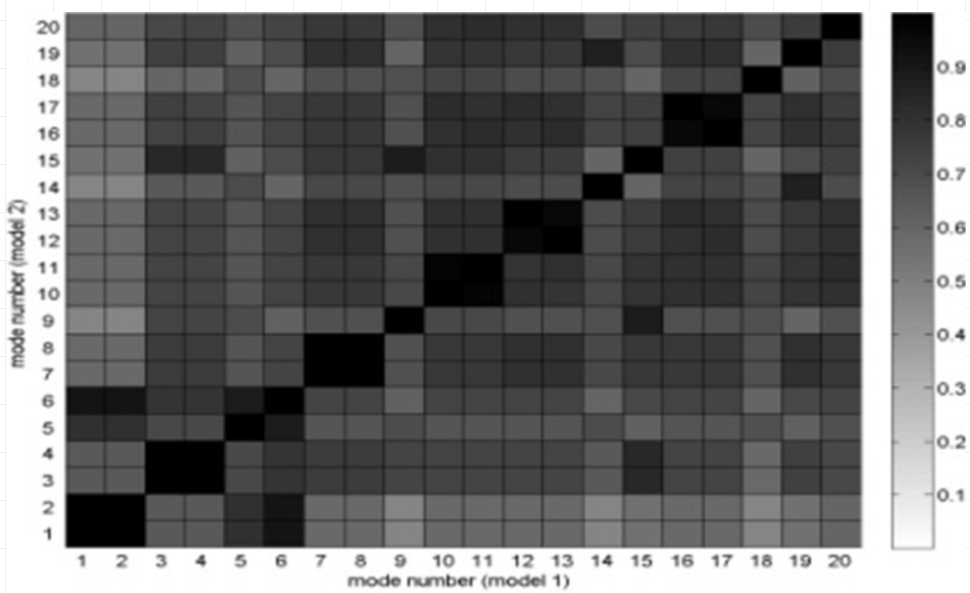


Fig. 8. MAC plot with % variation of E

Auto Mac will be the same in both the cases but MAC will be change when hairline crack has been introduced in the steel model then again same procedure adopted for the calculation of damage index. No Temperature effects were observed on MAC.

VIII. Frequency Comparison

A. Without crack model of steel

These are the plots between the mode shape number and the natural frequencies of aluminium and steel. These show that the natural frequency of aluminium is almost near to the natural frequency of the steel beam. The initial frequencies of aluminium and steel beams are almost same but when the mode shape increases than the negligible difference occurs between the natural frequencies of these two beams. Basically the initial mode shape show the global behaviour of the structure and the higher mode shapes shows the localized behaviour of the structure. So when the modulus of elasticity increases in the steel beam from the original steel then the natural frequency increases from the aluminium beam and when modulus of elasticity decreases from the original steel then the natural frequency will be decreases from the original aluminium which is best described in the higher mode shapes so the overall comparison is shown in Fig. 9.

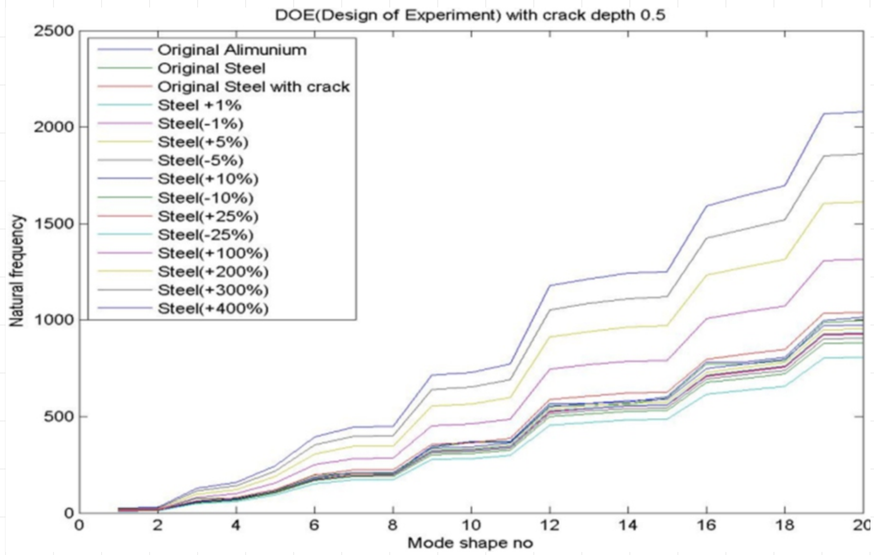


Fig. 9. Frequency based comparison of steel beam models (without crack)

In this analysis we observed that in the case of original aluminium and original steel the natural frequencies of these two beam structure have no significance difference because the natural frequency depends upon the k/m ratio of the structures. In the analysis we also observed that when the modulus of elasticity increases then the natural frequency of that structure increases from the aluminium structure which is clearly shown in the higher modes.

B. With Crack Model of Steel

These plot shows that the first two mode shapes having the same natural frequencies means that the crack has not effects on the first two mode shapes so when the crack propagation start then it highly effects on the frequencies of the mode shapes which is shown by the plot. Basically crack show the localized behaviour of the structure which is beat described by the higher mode shapes while the initial mode shapes show the global behaviour of the structure.

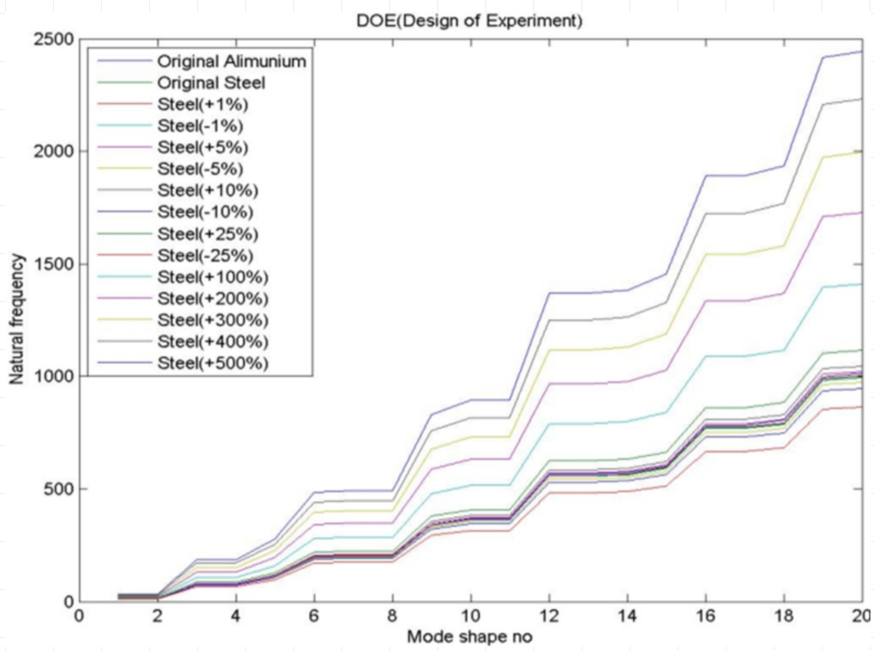


Fig. 10. Frequency based comparison of steel beam models (with crack)

In this analysis we observed that up to +10% increment the value of natural frequency are lower than the reference model but above the +10% increments the value of natural frequency become greater than the reference model which is best described by the higher mode shapes. In this analysis we observed that the initial modes of crack and without crack model have almost the same natural frequencies because the initial modes describe the global behaviour of the structure.

IX. MAC BASED CRACK IDENTIFICATION

In this research the mode shape matrix of first 20 modes of aluminium was compared with the mode shape matrix of steel beam structure which having the different modulus of elasticity. So from these comparisons of mode shape we obtained the different Mac in each case. While in this case the Automac is obtained by comparing the mode shape matrix of aluminium beam and steel beam which is taking as the reference. So from each Mac and Auto Mac we obtain the Mac and Automac matrices. By subtracting the Mac matrices from the Automac matrices we obtained the resultant matrix which indicates the change of Mac. So the change in Mac basically measures the disorder of the system due to the change in modulus of elasticity in the first case. So for obtaining the single numerical value of the resulting matrix, Frobeinus norm were introduced for calculating that value and that value basically measure the damage index or MAC severity (disorder of the system). Similarly for the case-2 when crack has been introduced in the steel beam alongwith the change in E of that structure then again the damage index obtained from the MAC and Automac.

Case-1

The table 04 shows that the value of damage index is different from the other one when only the modulus of elasticity changes in the steel beam.

TABLE IV
DAMAGE INDEX VALUES IN CRACK FREE STEEL BEAMS

WITHOUT CRACK				
Auto MAC (Reference)		MAC		Frob.norm/Damage index
Aluminium	Steel	Aluminium	Steel+1%	0.084932935
Aluminium	Steel	Aluminium	Steel-1%	0.103272701
Aluminium	Steel	Aluminium	Steel+5%	0.016982982
Aluminium	Steel	Aluminium	Steel-5%	0.018888895
Aluminium	Steel	Aluminium	Steel+10%	0.048004656
Aluminium	Steel	Aluminium	Steel-10%	0.040762473
Aluminium	Steel	Aluminium	Steel+25%	0.084867786
Aluminium	Steel	Aluminium	Steel-25%	0.058529253
Aluminium	Steel	Aluminium	Steel+100%	0.075285014
Aluminium	Steel	Aluminium	Steel+200%	0.019189406
Aluminium	Steel	Aluminium	Steel+300%	0.124669618
Aluminium	Steel	Aluminium	Steel+400%	0.050412813
Aluminium	Steel	Aluminium	Steel+500%	0.100284309

Case-2

In this case the crack has been introduced in the steel structure up to the crack depth ratio of 0.5 along with the changes in the modulus of elasticity which are mentioned in the table below. So according to these changes Mac is plotted but Auto Mac will be the same as in the case of previous one. Then damage index were obtained in each case by using the Mac and Automac. The damage indices for this case are shown in Table V.

TABLE V
DAMAGE INDEX VALUES IN STEEL BEAMS HAVING CRACK

STEEL WITH CRACK DEPTH (0.5)				
Auto MAC (Reference)		MAC		Frob.norm/Damage index
Aluminium	Steel	Aluminium	Steel+1%	2.052706216
Aluminium	Steel	Aluminium	Steel-1%	2.052706216
Aluminium	Steel	Aluminium	Steel+5%	2.052706216
Aluminium	Steel	Aluminium	Steel-5%	2.052706216
Aluminium	Steel	Aluminium	Steel+10%	2.052706216
Aluminium	Steel	Aluminium	Steel-10%	2.052706216
Aluminium	Steel	Aluminium	Steel+25%	2.052706216
Aluminium	Steel	Aluminium	Steel-25%	2.052706216
Aluminium	Steel	Aluminium	Steel+100%	2.052706216
Aluminium	Steel	Aluminium	Steel+200%	2.052706216
Aluminium	Steel	Aluminium	Steel+300%	2.052706216
Aluminium	Steel	Aluminium	Steel+400%	2.052706216
Aluminium	Steel	Aluminium	Steel+500%	2.052706216

X. CONCLUSION

Mistuning in any structure causes deviation in its operational behavior and its extend can be directly related to the mistuning level. In this work, model mistuning (in the form of variation in material properties) is investigates. It is found that with minute material variation, frobenius norm did not indicate any major change. Damage index was very low with small material variation. So it can be concluded that within material variations due to material manufacturing process has negligible impact on damage index. All such materials having sight variations in their properties can be considered close to the homogenous material. However, when this material change was higher in magnitude, damage index obtained by calculating frobenius norm indicated major change. Besides this, presence of damage in any structure ultimately changes its material properties which can be tracked through the method described in this work. The proposed vector correlation method in this research, work satisfactorily in identifying source of non-linearity (crack) in presence of material variation. Also this technique can be used to locate internal cracks in any structure.

XI. FUTURE RECOMMENDATIONS

Minimum material properties variation level, which results in changing damage index, is still needed to be investigated.

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